## *ALARCON*

### CHAPTER 1: LIMITS

EXERCISE 1.1

1. *I f* f *x* = *x* 2 – 4 *x*, *f ind a* f – 5 *b* f

*y* 2 + 1 *c f x* + *x d f x* + 1 – *f x* – 1

1. f – 1 =

= 25 + 20

f – 1 = 4 5

– 5 2 – 4 – 5

1. f *y* 2 + 1 =

*y* 2 + 1 2 – 4

*y* 2 + 1

= *y* 4 + 2 *y* 2 + 1 – 4 *y* 2 – 4

= *y* 4 – 2 *y* 2 – 3

f *y* 2 + 1 =

*y* 2 – 3

*y* 2 + 1

*c* f *x* +  *x* = *x* +  *x* 2 – 4 *x* +  *x*

= *x* +  *x x* +  *x* – 4

f *x* +  *x* = *x* +  *x x* +  *x* – 4

*d* f *x* + 1 – f *x* – 1 =

*x* + 1 2 – 4 *x* + 1 –

*x* – 1 2 – 4 *x* – 1

= *x* 2 + 2 *x* + 1 – 4 *x* – 4 – *x* 2 – 2 *x* + 1 – 4 *x* + 4

= *x* 2 – 2 *x* – 3 – *x* 2 – 6 *x* + 5

= *x* 2 – 2 *x* – 3 – *x* 2 + 6 *x* – 5

f *x* + 1 – f *x* – 1 = 4 *x* – 8 *or* 2 *x* – 4

= *x* 2 + 3

## *I f y x* , *f ind x as a function of y*

= *x* 2 + 3

*y* *x*

*x* 2 + 3 = *xy*

*x* 2 – *xy* + 3 = 0

Find the value of x by using the quadratic equation.

*x* = 2 *a*

– *b*  *b* 2 – 4 *ac*

, *a* =

1, *b*

= – *y*, *c* = 3

*x* =

– – *y*  – *y* 2 – 4 1 3

2 1

*y*  *y* 2 – 12

=

## *x*

2

3 If y= tan *x* + *π* , find x as a function of y.

*y* = tan *x* + *π*

*x* + *π* = arctan *y x* = arctan *y* – *π*

1. Express the distance D travaled in t hr by a car whose speed is 60 km/hr. Distance = Rate  Time

*D* = 60 *t*

1. Express the area A of an equilateral triangle as a function of its side x.

*A* = 1 *bh*

2

*x*

*x*

1 *x*

2

3

3

*a* 2 + *b* 2 = *c* 2

2

1

2

*x* + *h* 2 = *x* 2

Let b = x, h = *x*

2

*A* = 1 *x* 3 *x*

1 *x* 2 + *h* 2 = *x* 2

4

*h* 2 = *x* 2 – 1 *x* 2

4

4 *x* 2 – *x* 2

2 2

*A* = 3 *x* 2

4

*h* 2 =

4

1 *x* 1 *x*

2 = 3 *x* 2 2 2

*h*

4

*h* = *x*

3

2

1. The stiffness of a beam of rectangular cross section is proportional to the breadth and the cube of the depth. If the breadth is 20 cm, express the stiffness as a function of the depth.

Let S = stiffness, b = breadth, d = depth

*D*

*S* = *b d* 3

*S* = 20 *d* 3

## *d*

*b*

1. A right circular cylinder, radius of base x, height y, is incribed in a right circular cone, radius of base r and height h. Express y as function of x (r and h are constant)

Using ratio and proportion of two similar

triangle  *BCD* and  *ACE E*

*D*

*y*

*B*

*x*

*A*

*y* = *h*

*r* – *x r*

*r y* = *h r* – *x h*

*y* = *h r* – *x*

*r*

## *C*

1. If f *x*

= *x* 2 +

1, find

f *x* + *h* – f *x* , *h* s 0

*h*

*r*

f *x* + *h* – f *x* =

*x* + *h* 2 + 1 – *x* 2 + 1

*h h*

= *x* 2 + 2 *xh* + *h* 2 + 1 – *x* 2 – 1

*h*

= 2 *xh* + *h* 2

*h*

= *h* 2 *x* + *h h*

f *x* + *h* – f *x* = 2 *x* + *h*

*h*

1. If f 3 *x* 2 – 4 *x* + 1 , find f *h* + 3 – f 3 , *h* s 0

*h*

f *h* + 3 – f 3 =

*h h*

|  |  |  |
| --- | --- | --- |
| 3 *h* + 3 2 – 4 *h* + 3 + 1 | – | 3 3 2 – 4 3 + 1 |

=

*h*

|  |  |  |
| --- | --- | --- |
| 3 *h* 2 + 6 *h* + 9 – 4 *h* – 12 + 1 | – | 27 – 12 + 1 |

= 3 *h* 2 + 18 *h* + 27 – 4 *h* – 12 + 1 – 27 + 12 – 1

*h*

= 3 *h* 2 + 14 *h*

*h*

= *h* 3 *h* + 14

*h*

f *h* + 3 – f 3 = 3 *h* + 14

*h*

10 If f *x* = 4

+

*x* 3

f g *x* = 4

and g *x* = *x* 2 – 3, find f g *x* and g f *x*

*x* 2 – 3 + 3

f g *x* = 4

*x* 2

2

# g f *x*

= 4 – 3

*x* + 3

g f *x* = 16 3

*x* 2 + 6 *x* + 9

16 – 3 *x* 2 + 18 *x* + 27

g f *x* =

*x* 2 + 6 *x* + 9

g f *x*

g f *x*

g f *x*

= 16 – 3 *x* 2 – 18 *x* – 27

*x* 2 + 6 *x* + 9

= – 3 *x* 2 – 18 *x* – 11

*x* 2 + 6 *x* + 9

= – 3 *x* 2 – 18 *x* – 11

*x* + 3 2

### EXECRCISE 1.2

Evaluate each of the following.

# lim

*x*→2

*x* 2 – 4 *x* + 3

= 2 2 – 4 2 + 3

= 4 – 8 + 3

# lim

*x*→2

*x* 2 – 4 *x* + 3

= – 1

# lim

3 *x* + 2

*x* + 4

*x*→3

= 3 3 + 2

3 + 4

= 9 + 2

7

lim 3 *x* + 2

= 11

*x*→3

*x* + 4 7

1. lim tan *x* + sin *x*

*x*→ *π*

4

# = tan *π*

4

+ sin *π*

4

= 1 +

2

2

lim tan *x* + sin *x*

*x*→ *π*

4

sin 2 *x*

sin *x*

# lim

*x*→ *π*

3

sin 2 *π*

3

= 2 +

2

2

=

sin *π*

3

sin 2 *π*

= 3

sin *π*

3

3

= 2

2

3

lim sin 2 *x* = 1

*x*→ *π*

3

sin *x*

1. lim 2 *x* + 4

3

*x*

*x*→8

= 2 8 + – 4

3

8

= 16 + 2 – 4

3

*x*

lim 2 *x* +

*x*→8

– 4 = 14

1. lim 4 *x* – 3

*x*→2

= 4 2 – 3

= 8 – 3 4 + 5

= 5 9

*x* 2 + 5

2 2 + 5

lim 4 *x* – 3

*x*→2



# lim

= 45

3 *x*

+

*x*→3 *x x* 1

= 3 3

3 3 + 1

= 9

3 4

lim 3 *x* = 1

*x*→3 *x x* + 1 2

3 *x* + 2

# lim

*x*→0

*x* 2 – 2 *x* + 4

= 3 0 + 2

0 2 – 2 0 + 4

= 2

4

lim 3 *x* + 2 = 1

*x*→0 *x* 2 – 2 *x* + 4 2

### EXERCISE 1.3

Evaluate each of the following. 1.

*x*→4 *x* – 16

lim

*x* 3 – 64

2

*x* – 4 *x* 2 + 4 *x* + 16

*x* – 4 *x* + 4

=

*x* 2 + 4 *x* + 16

=

*x* + 4 *\*

= 4 2 + 4 4 + 16

4 + 4

= 16 + 16 + 16

8

= 48

8

# lim

*x*→4

*x* 3 – 64

*x* 2 – 16 6

=

# lim

*x*→2

*x* 2 + 2 *x* – 8

3 *x* – 6

= *x* + 4 *x* – 2 3 *x* – 2

= *x* + 4

3

= 2 + 4

3

= 6

3

# lim

*x*→2

*x* 2 + 2 *x* – 8

3 *x* – 6 2

=

# lim

*x*→3

= *x* – 3

*x* – 3

*x* 3 – 13 *x* + 12

*x* 3 – 14 *x* + 15

*x* 2 + 3 *x* – 4

*x* 2 + 3 *x* – 5

= *x* 2 + 3 *x* – 4

*x* 2 + 3 *x* – 5

3 2 + 3 3 – 4

3 2 + 3 3 – 5

=

= 9 + 9 – 4

9 + 9 – 5

*x* 3 – 13 *x* + 12

= 14

# lim

*x*→3

*x* 3 –

14 *x* + 15 13

*x* 3 – *x* 2 – *x* – 2

1. lim 3 2

*x*→2 2 *x* – 5 *x* + 5 *x* – 6

= *x* – 2 *x* 2 + *x* + 1

*x* – 2 2 *x* 2 – *x* + 3

= *x* 2 + *x* + 1 2 *x* 2 – *x* + 3

= 2 2 + 2 + 1

2 2 2 – 2 + 3

= 4 + 2 + 1

8 – 2 + 3

lim

*x* 3 – *x* 2 – *x* – 2

3 – 2 +

–

5. lim

*x*→2 2 *x* 5 *x* 5 *x* 6

*x* + 3 2 – 9

*x*→0 2 *x*

= 7

9

=

*x* + 3 2 – 3 2

2 *x*

= *x* + 3 – 3 *x* + 3 + 3

2 *x*

= *x x* + 6 2 *x*

= *x* + 6

2

= 6

2

# lim

*x*→0

*x* + 3 2 – 9

2 *x* 3

=

# lim

*x*→0

*x* + 16 – 4

*x*

= *x* + 16 – 4 \* *x* + 16 + 4

*x x* + 16 + 4

= *x* + 16 – 16

*x* + 4

*x* + 16

= *x*

*x* + 4

*x* + 16

= 1

*x* + 16 + 4

= 1

16 + 4

= 1

4 + 4

lim *x* + 16 – 4 = 1

*x*→0

# lim

*x* 8

*x* – 1

+ –

*x*→1 *x* 3 2

= *x* – 1 \* *x* + 3 + 2

*x* + 3 – 2 *x* + 3 + 2

= *x* – 1 *x* + 3 + 2

*x* + 3 – 4

= *x* – 1 + 2

*x* + 3

*x* – 1

= + 2

*x* + 3

= + 2

1 + 3

= + 2

4

= 2 + 2

lim *x* – 1 = 4

→8

m

*x* + 3 – 2

3 *x* – 2

*x* – 8

3 *x* 2 + 2 3 *x* + 4

*x* – 8

\*

3

*x* 2 + 2 *x* + 4

3

*x*→1

# li

*x*

=

3 *x* – 2

= *x* – 8

3 *x* 2

*x* – 8

= 1

3 *x* 2 + 2 3 *x* + 4

= 1

3

8 + 2 8 + 4

2

3

= 1

+ 2 3

*x* + 4

3 64 + 2 2 + 4

= 1

4 + 4 + 4

3 *x* – 2

*x* – 8

# lim = 1

*x*→8 12

1 – 1

*x* 4

*x* – 4

# lim

*x*→4

4 – *x*

= 4 *x*

*x* – 4

= 4 – *x*

4 *x x* – 4

= – *x* – 4

4 *x x* – 4

= – 1

4 *x*

= –



1

4 4

1 – 1

# lim *x*

4 = – 1

*x*→4 *x* – 4 16

*x* 3 – 8

# lim

*x*→2

= *x* – 2

*x* 2 – 4

*x* 2 + 2 *x* + 4

*x* – 2 *x* + 2

= *x* 2 + 2 *x* + 4

*x* + 2

= 2 2 + 2 2 + 4

2 + 2

= 4 + 4 + 4

4

= 12

4

4 – *x*

4 – *x*

2 *x* – 6

*x*→2 *x* – 4

lim

*x* 3 – 8

2

= 3

11. lim

*x* – 3

*x*→3

*x* 2 4 *x*

– – –

= *x* – 3 \* *x* – 2 +

*x* – 2 – 4 – *x x* – 2 +

= *x* – 3 *x* – 2 + 4 – *x*

*x* – 2 – 4 – *x*

= *x* – 3 *x* – 2 + 4 – *x*

2

= *x* – 3 *x* – 2 + 4 – *x*

2 *x* – 3

= *x* – 2 + 4 – *x*

3 – 2 + 4 – 3

=

2

= 1 + 1

= 2 2

2

lim *x* – 3 = 1

*x*→3 *x* – 2 – 4 – *x*

12. lim 1 1 – 1

*x*→0 *x* 3

*x* + 9

= 1

*x*

= 1 3 *x*

*x* + 9 – 3

3 *x* + 9

*x* + 9 – 3

*x* + 9

*x* + 9 + 3

*x* + 9 + 3

= 1 3 *x*

*x* + 9 – 9

*x* + 9 *x* + 9 + 3

=

1

*x*

3 *x x* + 9 + 3 *x* + 9

= *x*

*x* + 9

3 *x x* + 9 + 3

= 1

3 *x* + 9 + 9 *x* + 9

= 1

3 0 + 27 + 9 0 + 9

= 1

0 + 27 + 27

# lim 1

1 – 1 = 1

*x*→0 *x*

3 *x* + 9 54

*x* 2 – 9

# lim

*x* – 3

*x*→3

= *x* 2 – 9

\*

*x* – 3

*x* 2 – 9

*x* 2 – 9

= *x* 2 – 9

*x* – 3 *x* 2 – 9

*x* – 3 *x* + 3

*x* – 3 *x* 2 – 9

=

= *x* + 3

*x* 2 – 9

= 3 + 3

3 2 – 9

= 6

0

*x* 2 – 9

*x* – 3

# lim

*x*→3

tan 2 *x*

sec 2 *x*

# lim

*x*→ *π*

4

sin 2 *x*

= cos 2 *x* 1

cos 2 *x*

=  The Limit does not exist

= sin 2 *x* cos 2 *x*

cos 2 *x*

= sin 2 *x*

= sin 2 *π*

4

# = sin *π*

2

lim tan 2 *x* = 1

*x*→ *π*

4

sec 2 *x*

# lim

sin 3 *x*

sin *x* – tan *x*

*x*→0

= sin 3 *x*

– sin *x*

sin *x*

cos *x*

sin 3 *x*

=

sin *x* cos *x* – sin *x*

cos *x*

= sin 3 *x* cos *x*

sin *x* cos *x* – sin *x*

= sin 3 *x* cos *x*

sin *x* cos *x* – 1

= sin 2 *x* cos *x*

cos *x* – 1

= 1 – cos 2 *x* cos *x*

cos *x* – 1

= 1 – cos *x* 1 + cos *x* cos *x*

cos *x* – 1

= – 1 + cos *x* cos *x*

= – 1 + cos 0 cos 0

= –

1 + 1  1

sin 3 *x*

sin *x* – tan *x*

# lim

*x*→0

= – 2

# lim

*x*→0

1 – cos 2 *x*

1 + cos *x*

= 1 – cos *x* 1 + cos *x*

1 + cos *x*

= 1 – cos *x*

= 1 – cos 0

= 1 – 1

# lim

*x*→0

1 – cos 2 *x*

1 + cos *x* 0

=

# lim

*x*→0

sin *x* sin 2 *x*

1 – cos *x*

= sin *x* 2 sin *x* cos *x* 1 – cos *x*

= 2 sin 2 *x* cos *x*

1 – cos *x*

= 2 cos *x* 1 – cos 2 *x*

1 – cos *x*

= 2 cos *x* 1 – cos *x* 1 + cos *x*

1 – cos *x*

= 2 cos *x* 1 + cos *x*

= 2 cos 0 1 + cos 0

= 2 1 1 + 1

= 2 2

lim sin *x* sin 2 *x* = 4

*x*→0 1 – cos *x*

= 1 – cos 2 *x*

1 + cos *x*

= 1 – cos *x* 1 + cos *x*

1 + cos *x*

= 1 – cos *x*

= 1 – cos *π*

= 1 – – 1

sin 2 *x*

*x*→*π*

lim

1 cos *x*

+

= 2

If f *x* = *x* , find;

f *x* – f 4

# lim

*x*→4

*x* – 4

= *x* – 4

*x* – 4

= *x* – 2

*x* – 4

= *x* – 2  *x* + 2

*x* – 4 *x* + 2

= *x* – 4

*x* – 4 + 2

*x*

= 1

*x* + 2

= 1

4 + 2

= 1

2 + 2

lim f *x* – f 4 = 1

*x*→4 *x* – 4

4

20. lim

*x*→0

f 9 + *x* – f 9

*x*

= 9 + *x* – 9

*x*

= *x* + 9 – 3

*x*

= *x* + 9 – 3  *x* + 9 + 3

*x x* + 9 + 3

= *x* + 9 – 9

*x* + 3

*x* + 9

= *x*

*x* + 3

*x* + 9

= 1

*x* + 9 + 3

= 1

0 + 9 + 3

= 1

9 + 3

= 1

3 + 3

lim f 9 + *x* – f 9 = 1

*x*→0 *x* 6

If f *x* = *x* 2 – 2 *x* + 3, find;

f *x* – f 2

*x* – 2

# lim

*x*→2

= *x* – 2

*x* 2 – 2 *x* + 3 – 2 2 – 2 2 + 3

= *x* 2 – 2 *x* + 3 – 4 + 4 – 3

*x* – 2

= *x* 2 – 2 *x*

*x* – 2

= *x x* – 2

*x* – 2

= *x*

lim f

*x* – f 2

*x* – 2

= 2

f *x* + 2 – f 2

*x*

*x*→2

# lim

*x*→0

= *x*

= *x*

*x* 2 + 4 *x* + 4 – 2 *x* – 4 + 3 – 4 + 4 – 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* + 2 2 – 2 *x* + 2 + 3 | – | 2 | 2 – 2 | 2 | + 3 |

= *x* 2 + 2 *x x*

= *x x* + 2

*x*

= *x* + 2

= 0 + 2

lim f *x* + 2 – f 2 = 2

*x*→0 *x*

### EXERCISE 1.4

Evaluate each of the following.

6 *x* 3 + 4 *x* 2 + 5

8 *x* 3 + 7 *x* – 3

# lim

*x*→

### REMINDER:

lim 1 = 0

1

= 6 *x* 3 + 4 *x* 2 + 5  *x* 3

*x*→ *x*

8 *x* 3 + 7 *x* – 3 1

*x* 3

6 *x* 3 + 4 *x* 2 + 5

= *x* 3 *x* 3 *x* 3

8 *x* 3 + 7 *x* – 3

*x* 3 *x* 3 *x* 3

6 + 4 + 5

= *x x* 3

8 + 7 – 3

*x* 2 *x* 3

= 6 + 0 + 0

8 + 0 – 0

= 6

8

# lim = 3

6 *x* 3 + 4 *x* 2 + 5

8 *x* 3 + 7 *x* – 3

*x*→ 4

3 *x* 2 + *x* + 2

*x* 3 + 8 *x* + 1

# lim

*x*→

1

= 3 *x* 2 + *x* + 2  *x* 3

*x* 3 + 8 *x* + 1 1

*x* 3

3 *x* 2 + *x* + 2

= *x* 3 *x* 3 *x* 3

*x* 3 + 8 *x* + 1

*x* 3 *x* 3 *x* 3

3 + 1 + 2

= *x x* 2 *x* 3

1 + 8 + 1

*x* 2 *x* 3

= 0 + 0 + 0

1 + 0 + 0

# lim

*x*→

3 *x* 2 + *x* + 2

*x* 3 + 8 *x* + 1 0

=

4 *x* + 5

# lim

*x*→

*x* 2 + 1

1

= 4 *x* + 5  *x* 2

*x* 2 + 1 1

*x* 2

4 *x* + 5

= *x* 2 *x* 2

*x* 2 + 1

*x* 2 *x* 2

4 + 5

= *x x* 2

1 + 1

*x* 2

= 0 + 0

1 + 0

lim 4 *x* + 5 = 0

*x*→ *x* 2 + 1

*x* 3 + *x* + 2

*x* 2 – 1

# lim

*x*→

1

= *x* 3 + *x* + 2  *x* 3

*x* 2 – 1 1

*x* 3

*x* 3 + *x* + 2

= *x* 3

*x* 3 *x* 3

*x* 2 – 1

*x* 3 *x* 3

1 + 1 + 2

= *x* 2 *x* 3

1 – 1

*x x* 3

= 1 + 0 + 0

0 – 0

*x* 3 + *x* + 2 =

# lim

*x*→

# lim

*x*→

*x* 2 – 1 

8 *x* – 5

4 *x* 2 + 3

1

= 8 *x* – 5  *x*

4 *x* 2 + 3

1

*x*

8 *x* – 5

= *x x*

4 *x* 2 + 3

*x* 2 *x* 2

8 – 5

= *x*

4 + 3

*x* 2

= 8 – 0

4 + 0

= 8

2

lim 8 *x* – 5 = 4

4 *x* 2 + 3

*x*→

# lim

*x* 3

2 *x* – 1 2

*x*→

= *x*

3

4 *x* 2 – 4 *x* + 1

= *x* 3

4 *x* 2 – 4 *x* + 1

*x* 3

= *x* 3

1

· *x* 3

1

*x* 3

4 *x* 2 – 4 *x* + 1

*x* 3 *x* 3 *x* 3

= 1

4 – 4 + 1

*x x* 2 *x* 3

= 1

0 + 0 + 0

*x* 2

*x*→

lim

*x* 3

=

2 *x* – 1

2



7. lim

*x* + 3 3 – *x* – 2 3

*x*→

*x*

2

*x* + 9 *x* + 27 *x* + 27 – *x* – 6 *x* + 12 *x* – 8

3 2

3 2

=

= *x* 3 + 9 *x* 2 + 27 *x* + 27 – *x* 3 + 6 *x* 2 – 12 *x* + 8

*x* 2

= 15 *x* 2 + 15 *x* + 35

*x* 2

1

= 15 *x* 2 + 15 *x* + 35  *x* 2

*x* 2 1

*x* 2

15 *x* 2 + 15 *x* + 35

= *x* 2

*x* 2 *x* 2

*x* 2

*x* 2

15 + 15 + 35

= *x x* 2

1

= 15 + 0 + 0

1

= 15

*x*→

lim

8. lim

*x*→

*x* + 3 3 – *x* – 2 3

*x*

9 *x* 2 + 4

6 *x* 1

2

+

1

9 *x* 2 + 4

=  *x*

6 *x* + 1 1

*x*

9 *x* 2 + 4

*x* 2 *x* 2

=

6 *x* + 1

*x x*

9 + 4

*x* 2

=

6 + 1

*x*

9 + 0

= 6 + 0

= 3

6

9 *x* 2 + 4

6 *x* + 1

# lim = 1

*x*→ 2

### EXERCISE 1.5

Find the value or values of x for which the function is discontinuous. (The function is discontinuous if the denominator is equal to 0)

3 *x*

1. *x* – 5

*x* – 5 = 0

*x* = 5

Check

3 *x*

*x* – 5

= 3 5

5 – 5

= 15 , Therefore, the function is discontinuous if x=5.

0

3 *x* + 2

2. *x* 2 – 8 *x* + 15

*x* 2 – 8 *x* + 15 = 0

*x* – 5 *x* – 3 = 0

*x* = 5, *x* = 3

Check

3 5 + 2

5 2 – 8 5 + 15

= 17

0

3 3 + 2

3 2 – 8 3 + 15

= 11

0

Therefore, the function is discontinuos if x=5 and x=3.

5 *x* + 1

3. *x* 2 + 4

None. The function is continuous.

6 *x*

4. *x* 2 – 9

*x* 2 – 9 = 0

*x* – 3 *x* + 3 = 0

*x* = 3, *x* = – 3

Check

6 *x*

*x* 2 – 9

= 6  3

 3 2 – 9

=  18 0

Therefore, the function is discontinuous if *x* =  3.

1

5.

*x*

2 – 8

2 *x* – 8 = 0

2 *x* = 8

log2 8 = *x*, Use calculator or just simply assume a number that makes the denominator zero.

*x* = 3 Check 1

2 *x* – 8

= 1

2 3 – 8

= 1

8 – 8

log2 8 = *x*

log 2 3 = *x*, log *a r* = *r*

2 *a*

log 2 3 = 3

2

= 1 , Therefore, the function is discontinuos if x=3.

0

*x* + 3

6. *x* 3 – 3 *x* 2 + 2 *x*

*x* 3 – 3 *x* 2 + 2 *x* = 0

*x x* 2 – 3 *x* + 2 = 0

*x x* – 2 *x* – 1 = 0

*x* = 0, *x* = 2, *x* = 1

Check

*x* + 3

*x* 3 – 3 *x* 2 + 2 *x*

= 0 + 3

0 3 – 3 0 2 + 2 0

f 0 = 3

0

f 2 = 5

0

f 1 = 4

0

### EXERCISE 1.6

Sketch the graph of the following functions:

1. *y*

= 4 2. *y*

*x*

= 6

*x* 2

3. *y*

*x* 2 – 1

*x* 2 + 1 4. *y*

=

= 2 *x x* – 1

5. *y* =

*x* 2

*x* – 2 6. *y*

= 2 *x* 7. *y*

*x* 2 – 4

= 2 8. *y*

*x* 2 – 1

= *x* – 2

*x* 2 – 9 *x* + 20

9. *y*

= 4 *x* 10. *y*

*x* 2 – 9

= *x* 2 + 1

*x*

### CHAPTER 2: DIFFERENTIATION OF ALGEBRAIC FUNCTIONS

EXERCISE 2.1

Find the derivative by use of Delta Method. 1. *y* = 4 *x* 2 – 5 *x*

*y* +  *y* = 4 *x* +  *x* 2 – 5 *x* +  *x*

 *y* =

4 *x* +  *x* 2 – 5 *x* +  *x*

– 4 *x* 2 – 5 *x*

d *y* = lim  *y*

d *x* *x*→0  *x*

= 4 *x* +  *x* 2 – 5 *x* +  *x*

 *x*

– 4 *x* 2 – 5 *x*

= 4 *x* 2 + 2 *x*  *x* +  *x* 2

– 5 *x* – 5  *x* – 4 *x* 2 + 5 *x*

 *x*

= 4 *x* 2 + 8 *x*  *x* + 4  *x* 2 – 5 *x* – 5  *x* – 4 *x* 2 + 5 *x*

 *x*

= 8 *x*  *x* + 4  *x* 2 – 5  *x*

 *x*

=  *x* 8 *x* + 4  *x* – 5

 *x*

= lim 8 *x* + 4  *x* – 5

*x*→0

= lim 8 *x* + 4 0 – 5

*x*→0

d *y* = 8 *x* – 5

d *x*

2. *y* = *x* 3 + 2 *x*

*y* +  *y* = *x* +  *x* 3 + 2 *x* +  *x*

 *y* = *x* +  *x* 3 + 2 *x* +  *x*

d *y* = lim  *y*

– *x* 3 + 2 *x*

d *x* *x*→0  *x*

= *x* +  *x* 3 + 2 *x* +  *x*

 *x*

– *x* 3 + 2 *x*

= *x* 3 + 3 *x* 2  *x* + 3 *x*  *x* 2 +  *x* 3

 *x*

+ 2 *x* + 2  *x* – *x* 3 – 2 *x*

= *x* 3 + 3 *x* 2  *x* + 3 *x*  *x* 2 +  *x* 3 + 2 *x* + 2  *x* – *x* 3 – 2 *x*

 *x*

= 3 *x* 2  *x* + 2 *x*  *x* 2 +  *x* 3 + 2  *x*

 *x*

=  *x* 3 *x* 2 + 2 *x*  *x* +  *x* 2 + 2

 *x*

= lim 3 *x* 2 + 2 *x*  *x* +  *x* 2 + 2

*x*→0

= lim 3 *x* 2 + 2 *x* 0 +

*x*→0

d *y* = 3 *x* 2 + 2

d *x*

3. *y* = 4

*x*

*y* +  *y* = 4

*x* +  *x*

0 2 + 2

 *y* = 4 4

*x* +  *x*

*x*

d *y* = lim

 *y*

 *x*

d *x* *x*→0

–

*x* +  *x*

*x*

4

4

=  *x*

– +

*x* +  *x*

*x*

*x* +  *x*

*x* +  *x*

*x*

*x*

4

4

4

4

=  *x*  4 + 4

= 16 *x* +  *x* – 16 *x*

*x*

 *x* 4 *x* +  *x* + 4

= 16 *x* + 16  *x* – 16 *x*

 *x* 4 + 4

*x* +  *x*

*x*

= 16  *x*

 *x* 4 *x* +  *x* + 4

16

*x*→0 4 *x* +  *x* + 4

lim

*x*

*x*

=

# = lim

16

4 *x* + 0 + 4 *x*

*x*→0

# = lim



16

8 *x*

*x*→0

d *y* = 2



*x*

d *x*

4. *y* = 6

*x*

+ = 6

*y*  *y*

*x* +  *x*

= 6 – 6

 *y*

*x* +  *x x*

 *y*

 *x*

d *y* = lim

d *x* *x*→0

6 – 6

= *x* +  *x x*

 *x*

6 *x* – 6 *x* +  *x*

= *x x* +  *x*

 *x*

= 6 *x* – 6 *x* – 6  *x x*  *x x* +  *x*

= – 6  *x x*  *x x* +  *x*

= lim – 6

*x*→0 *x x* +  *x*

= lim – 6

*x*→0 *x x* + 0

d *y* = – 6

*x* 2

3

*x*

d *x*

5. *y* =

*y* +  *y* =

3

*x* +  *x*

 *y* = –

3

*x* +  *x*

3

*x*

d *y* = lim

 *y*

 *x*

d *x* *x*→0

3 *x* +  *x* – 3 *x*

=  *x*

3 *x* +  *x* – 3 *x*

3

3

*x* +  *x* 2 +

*x* +  *x* 2 +

+ 3 *x* 2

+ 3 *x* 2

3 *x x* +  *x*

3 *x x* +  *x*

=  *x* 

=  *x*

3 *x x* +  *x*

3 *x* 2

 *x* 3 *x* +  *x* 2 + +

# = lim

*x*→0 3

# = lim

1

*x* +  *x* 2 +

3 *x x* +  *x*

1

+ 3 *x* 2

*x*→0 3

*x* + 0 2 + 3 *x x* + 0

+ 3 *x* 2

# = lim 1

*x*→0

3 *x* 2 + 3

*x* 2 + 3 *x* 2

d *y* = 1

3 3 *x* 2

d *x*

6. *y* = 2 – 5 *x*

*y* +  *y* = 2 – 5 *x* +  *x*

 *y* = 2 – 5 *x* +  *x* – 2 – 5 *x*

d *y* = lim  *y*

d *x* *x*→0  *x*

= 2 – 5 *x* +  *x* – 2 – 5 *x*

 *x*

= 2 – 5 *x* – 5  *x* – 2 + 5 *x*

 *x*

= – 5  *x*

 *x*

d *y* = – 5

d *x*

7. *y* =

4 *x* + 3

*y* +  *y* =

4 *x* +  *x* + 3

 *y* = –

*x*→0

lim

4 *x* +  *x* + 3

 *y*

 *x*

4 *x* + 3

d *y* =

d *x*

= 4 *x* +  *x* + 3 –

4 *x* + 3

 *x*

= 4 *x* + 4  *x* + 3 – 4 *x* – 3

· 4 *x* +  *x* + 3 +

4 *x* +  *x* + 3 +

4 *x* + 3

4 *x* + 3

 *x* +

4 *x* +  *x* + 3

4 *x* + 3

= 4  *x*

4 *x* + 3

 *x* 4 *x* +  *x* + 3 +

[= lim 4](#_TOC_250002)

*x*→0 4 *x* +  *x* + 3 + 4 *x* + 3

[= lim 4](#_TOC_250001)

x→0 4 x+ 0 + 3 + 4 x+ 3

[= lim 4](#_TOC_250000)

*x*→0 2 4 *x* + 3

d *y* = 2

4 *x* + 3

d *x*

8. *y* = 2 *x*

+

*x* 1

+ = 2 *x* +  *x x* +  *x* + 1

*x*

*y*  *y*

 *y* = 2 +  *x* – 2 *x*

*x* +  *x* + 1 *x* + 1

d *y* = lim  *y*

d *x* *x*→0  *x*

2 *x* +  *x* – 2 *x*

= *x* +  *x* + 1 *x* + 1

 *x*

= 2 *x* +  *x x* + 1 – 2 *x x* +  *x* + 1

 *x x* +  *x* + 1 *x* + 1

= 2 *x* 2 + *x* + *x*  *x* +  *x* – 2 *x* 2 – 2 *x*  *x* – 2 *x*

|  |  |  |
| --- | --- | --- |
|  *x* | *x* +  *x* + 1 | *x* + 1 |

= 2 *x* 2 + 2 *x* + 2 *x*  *x* + 2  *x* – 2 *x* 2 – 2 *x*  *x* – 2 *x*

|  |  |  |
| --- | --- | --- |
|  *x* | *x* +  *x* + 1 | *x* + 1 |

= 2  *x*

 *x x* +  *x* + 1 *x* + 1

# = lim 2

*x*→0

# = lim

*x*→0

# = lim

*x*→0

d *y* =

d *x*

1. *y* =

*y* +  *y* =

 *y* =

*x*→0

lim

2 *x* +  *x* + 1

 *y*

 *x*

d *y* =

d *x*

*x* +  *x* + 1

2

*x* + 0 + 1

2

*x* + 1 *x* + 1

2

*x* + 1 2

3

2 *x* + 1

3

2 *x* +  *x* + 1

3

*x* + 1

*x* + 1

– 3

2 *x* + 1

3 – 3

= 2 *x* +  *x* + 1

2 *x* + 1

 *x*

= 3 2 *x* + 1 – 3 2 *x* +  *x* + 1

 *x* 2 *x* + 1 2 *x* +  *x* + 1

= 3 2 *x* + 1 – 3 2 *x* +  *x* + 1  3 2 *x* + 1 + 3

2 *x* +  *x* + 1

2 *x* +  *x* + 1

 *x* 2 *x* + 1 2 *x* +  *x* + 1 3 2 *x* + 1 + 3

= 9 2 *x* + 1 – 9 2 *x* +  *x* + 1

 *x* 2 *x* + 1 2 *x* +  *x* + 1 3 2 *x* + 1 + 3

2 *x* +  *x* + 1

= 18 *x* + 9 – 18 *x* – 18  *x* – 9

 *x* 3 + 3

2 *x* + 1 2 *x* +  *x* + 1

2 *x* + 1

2 *x* +  *x* + 1

= – 18  *x*

 *x* 3 + 3

2 *x* + 1 2 *x* +  *x* + 1

2 *x* + 1

2 *x* +  *x* + 1

= lim – 18

2 *x* + 1

+ 3 2 *x* +  *x* + 1

3 2 *x* + 0 + 1

*x*→0

2 *x* + 1 2 *x* +  *x* + 1 3

= lim – 18

*x*→0 2 *x* + 1 2 *x* + 0 + 1 +

3 2 *x* + 1

2 *x* + 1

= lim – 18

*x*→0 2 *x* + 1 2 *x* + 1 6

= lim – 18

2 *x* + 1

*x*→0 6

2 *x* + 1

d *y* = – 3

2 *x* + 1 2 *x* + 1

d *x*

## *y*

= 5 *x* 2

4 *x* – 1

=  *x*

*y* +  *y* =

5 *x* +  *x* 2

4 *x* +  *x* – 1

 *y* =

5 *x* +  *x* 2 5 *x* 2

4 *x* +  *x* – 1 4 *x* – 1

–

d *y* = lim

 *y*

d *x*

*x*→0  *x*

4 *x* +  *x* – 1 – 4 *x* – 1

5 *x* +  *x* 2 5 *x* 2

|  |  |  |  |
| --- | --- | --- | --- |
| 5 *x* +  *x* 2 | 4 *x* – 1 – 5 *x* 2 | | 4 *x* +  *x* – 1 |
|  *x* | 4 *x* +  *x* – 1 |  | |

=

=

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 5 *x* 2 + 10 *x*  *x* + 5  *x* 2 | | 4 *x* – 1 – 5 *x* 2 | | 4 *x* + 4  *x* – 1 |
|  *x* | 4 *x* +  *x* – 1 | |  | |

4 *x* – 1

4 *x* – 1

= 20 *x* 3 – 5 *x* 2 + 40 *x* 2  *x* – 10 *x*  *x* + 20 *x*  *x* 2 – 5  *x* 2 – 20 *x* 3 – 20 *x* 2  *x* + 5 *x* 2

 *x* 4 *x* +  *x* – 1 4 *x* – 1

= 20 *x* 2  *x* – 10 *x*  *x* + 20 *x*  *x* 2 – 5  *x* 2

|  |  |  |
| --- | --- | --- |
|  *x* | 4 *x* +  *x* – 1 | 4 *x* – 1 |

|  |  |  |
| --- | --- | --- |
|  *x* | 4 *x* +  *x* – 1 | 4 *x* – 1 |

=  *x* 20 *x* 2 – 10 *x* + 20 *x*  *x* – 5  *x*

# = lim

*x*→0

# = lim

*x*→0

# = lim

*x*→0

20 *x* 2 – 10 *x* + 20 *x*  *x* – 5  *x*

|  |  |  |
| --- | --- | --- |
|  | 4 *x* +  *x* – 1 | 4 *x* – 1 |

20 *x* 2 – 10 *x* + 20 *x* 0 – 5 0

|  |  |  |
| --- | --- | --- |
|  | 4 *x* + 0 – 1 | 4 *x* – 1 |

20 *x* 2 – 10 *x*

4 *x* – 1 4 *x* – 1

d *y* = 10 *x* 2 *x* – 1

d *x* 4 *x* – 1 2

1. Given *s* = *t* , find d *s*

d *t*

*s* +  *s* =

*t* +  *t*

 *s* = –

*t* +  *t*

*t*

d *s* = lim

 *s*

 *t*

d *t* *t*→0

= *t* +  *t* – *t*

 *t*

*t*

= *t* +  *t* –

 *t*

· *t* +  *t* +

*t* +  *t* +

*t*

*t*

= *t* +  *t* – *t*

 *t* +

*t* +  *t*

*t*

=  *t*

 *t t* +  *t* +

= lim

*t*→0

= lim

*t*→0

1

*t* +  *t* + *t*

1

*t* + 0 + *t*

*t*

d *s* = 1



2 *t*

d *t*

1. Given *A* = *r* 2 find d *A*

*π* ,

d *r*

*A* +  *A* = *π r* +  *r* 2

 *A* = *π r* +  *r* 2 – *π r* 2

d *A* = lim  *A*

d *r* *r*→0  *r*

= *π r* +  *r* 2 –*π r* 2

 *r*

= *π r* 2 + 2 *r*  *r* +  *r* 2

 *r*

–*π r* 2

= *π r* 2 + 2*π r*  *r* +*π*  *r* 2 –*π r* 2

 *r*

= 2*π r*  *r* +*π*  *r* 2

 *r*

=  *r* 2*π r* +*π*  *r*

 *r*

= lim 2 *π r* + *π*  *r*

*r*→0

= lim 2 *π r* + *π* 0

*r*→0

d *A* = 2 *r*

*π*

d *r*

1. Given *V* = 4 *π r* 3 , find d *V*

*V* +  *V* =

 *V* =

3 d *r*

4*π r* +  *r* 3

3

4*π r* +  *r* 3

– 4 *π r* 3

3 3

=  *r*

12*π r* +  *r* 3 – 12*π r* 3

3

= 9  *r*

4 *π r* 3

3

d *V* = lim

 *V*

d *r*

*r*→0  *r*

4 *π r* +  *r* 3

–

= 12*π*

*r* 3 + 3 *r* 2  *r* + 3 *r*  *r* 2 +  *r* 3

9  *r*

– 12*π r* 3

= 12*π r* 3 + 36*π r* 2  *r* + 36*π r*  *r* 2 + 12  *r* 3 – 12*π r* 3

9  *r*

=

3  *r* 12*π r* 2 + 12*π r*  *r* + 4  *r* 2

# = lim

12*π r* 2 + 12*π r*  *r* + 4  *r* 2

3

12*π r* 2 + 12*π r* 0 + 4 0

3

12*π r* 2

3

*r*→0

# = lim

*r*→0

# = lim

*r*→0

9  *r*

d *V* = 4 *r* 2

*π*

d *r*

1. Given *S* = 4 *r* 2 find d *S*

*π* ,

d *r*

*S* +  *S* = 4*π r* +  *r* 2

 *S* = 4*π r* +  *r* 2 – 4*π r* 2

d *S* = lim  *S*

d *r* *r*→0  *r*

= 4*π*

= 4*π*

*r* +  *r* 2 – 4*π r* 2

 *r*

*r* 2 + 2 *r*  *r* +  *r* 2

 *r*

– 4*π r* 2

= 4*π r* 2 + 8*π r*  *r* + 4*π*  *r* 2 – 4*π r* 2

 *r*

=  *r* 8*π r* + 4*π*  *r*

 *r*

= lim 8 *π r* + 4*π*  *r*

*r*→0

= lim 8 *π r* + 4*π* 0

*r*→0

d *S* = 8 *r*

*π*

d *r*

1. Given *S* = 2 *t* + 3 , find d *S*

3 *t* – 4 d *t*

+ = 2 *t* +  *t* + 3 3 *t* +  *t* – 4

*S*  *S*

 *S* = 2 *t* +  *t* + 3 – 2 *t* + 3

3 *t* +  *t* – 4 3 *t* – 4

d *S* = lim  *S*

d *t* *r*→0  *r*

2 *t* +  *t* + 3 – 2 *t* + 3

= 3 *t* +  *t* – 4 3 *t* – 4

 *t*

|  |  |  |  |
| --- | --- | --- | --- |
| 2 *t* +  *t* + 3 | 3 *t* – 4 – 2 *t* + 3 | | 3 *t* +  *t* – 4 |
|  *t* | 3 *t* +  *t* – 4 |  | |

= 3 *t* – 4

= 2 *t* + 2  *t* + 3 3 *t* – 4 – 2 *t* + 3 3 *t* + 3  *t* – 4

 *t* 3 *t* +  *t* – 4 3 *t* – 4

= 6 *t* 2 – 8 *t* + 6 *t*  *t* – 8  *t* + 9 *t* – 12 – 6 *t* 2 – 9 *t* – 6 *t*  *t* – 9  *t* + 8 *t* + 12

 *t* 3 *t* +  *t* – 4 3 *t* – 4

= – 17  *t*

 *t* 3 *t* +  *t* – 4 3 *t* – 4

= lim – 17

*t*→0 3 *t* +  *t* – 4 3 *t* – 4

= lim – 17

*t*→0 3 *t* + 0 – 4 3 *t* – 4

= lim – 17

*t*→0 3 *t* – 4 3 *t* – 4

d *S* = – 17

d *t* 3 *t* – 4 2

### EXERCISE 2.2

d *y*

Find d *x* of each of the following;

1. *y* = 5 *x* 3 – 4 *x* 2 + 3 *x* – 6

Use power rule where; d *un*

d *x*

= *n un* – 1 d *u*

d *x*

and the constant rule where; d *c* = 0

d *x*

d *y* = 5 3

d *x*

*x* 3 – 1 – 4 2

*x* 2 – 1 + 3 1

*x* 1 – 1 – 0

d *y* = 15 *x* 2 – 8 *x* + 3

d *x*

3

*x*

*x*

1. *y* =

+ 4 +

Simplify *y* =

+ 4 *x* – 1 +

d

*n*

*x*

3

*x*

– d *u*  d

*m*

*n*  *d u*

Use power rule where; d *x u*

*x*

*n*

= *n un* 1

d *x*

or d *x u* = *m*

*n* – 1

*u m*

. du means the derivative of u.

d *y* 1

=

+ 4 – 1

*x* – 1 – 1 + 1

d *x* 1 – 1

3 *x* 3

1 – 1

2 *x* 2

d *y* = 1

3 3 *x* 2

5 – 6 *x*

d *x*

1. *y* =

– 4 + 1

*x* 2



2 *x*

Use power rule where; d = *d u*

*u*



2 *u*

d *x*

*u* = 5 – 6 *x d u* = – 6 *dx*

d *y* = – 6

2 5 – 6 *x*

d *x*

d *y* = 3

5 – 6 *x*

d *x*

1. *y* =

3

2 *x* – 7

Use power rule where;

*u* = 2 *x* – 7

*d u* = 2 *dx*

*n* = 1

*n*

d *u m*

d *x*

= *n m*

*d u*

*n* – 1 .

*u m*

*m* 3

d *y* = 2

3 3

2 *x* – 7 2

d *x*

5. *y* = 3 *x* 2 – 4 *x* + 1 5

Use chain rule where; d

d *x*

*u n* = *ndu*

*u n* – 1

*u* = 3 *x* 2 – 4 *x* + 1

*d u* = 6 *x* – 4 *dx*

d *y* = 5 6 *x* – 4

d *x*

3 *x* 2 – 4 *x* + 1 4

d *y* = 30 *x* – 20

d *x*

6. *y* =

7 + 3 *x* + 1

3 *x* 2 – 4 *x* + 1 4

Use chain rule where; d

d *x*

*u n* = *ndu*

*u n* – 1

and power rule where; d = *d u*

d *x*

*u*



2 *u*

*u* = 7 + 3 *x* + 1

*d u* = 3 *dx*

2 3 *x* + 1

3

d *y* =

2 3 *x* + 1

2 7 + 3 *x* + 1

d *x*

d *y* = 3

2 7 + 3 *x* + 1 2 3 *x* + 1

d *x*

d *y* = 3

4 3 *x* + 1 7 + 3 *x* + 1

d *x*

7. *y* = 4 *x* – 5

+

2 *x* 1

Use quotient rule where, d *u*

= *v du* – *udv* .

u = 4x - 5 du = 4dx

v = 2x + 1

dv = 2dx

2 *x* + 1  4 – 4 *x* – 5  2

2 *x* + 1 2

d *y* =

d *x*

d *y* = 8 *x* + 4 – 8 *x* + 10

d *x v v* 2

d *x* 2 *x* + 1 2

d *y* = 14

d *x* 2 *x* + 1 2

8. *y* = 3 *x* + 1

3 *x* 2 + 2

Use quotient rule where; d *u*

*u*

= *v du* – *udv*

and power rule where; d

= *d u*



2 *u*

*u* = 3 *x* + 1

*d u* = 3 *dx v* =

3 *x* 2 + 2

*d v* = 3 *x dx*

3 *x* 2 + 2

3 *x* 2 + 2 3 – 3 *x* + 1

3 *x*

3 *x* 2 + 2

3 *x* 2 + 2

2

3 3 *x* 2 + 2 – 9 *x* + 3 *x*

2

3 *x* 2 + 2

d *x v v* 2 d *x*

d *y* =

d *x*

d *y* =

d *x*

d *y* =

d *x*

d *y* =

d *x*

3 *x* 2 + 2

3 3 *x* 2 + 2 3 *x* 2 + 2 – 9 *x* 2 + 3 *x*

3 *x* 2 + 2 3 *x* 2 + 2

3 3 *x* 2 + 2 – 9 *x* 2 + 3 *x*

3 *x* 2 + 2

3

2

d *y* = 9 *x* 2 + 6 – 9 *x* 2 – 3 *x*

3

2

d *x*

3 *x* 2 + 2

d *y* = 6 – 3 *x*

3 *x* 2 + 2 3

d *x*

9. *y* = 2 *x* + 5

4 *x* – 1

Use product rule where; d *u v*

d *x*

= *udv* + *vdu* and power rule where; d

d *x*

= *d u*

*u* = 2 *x* + 5

*u*

*d u* = 2 *dx*

*v* = 4 *x* – 1

*d v* = 2

4 *x* – 1

d *y* = 2 *x* + 5 2 + 2



2 *u*

4 *x* – 1

d *x*

d *y* = 2 2 *x* + 5

4 *x* – 1

+ 2 4 *x* – 1

d *x* 4 *x* – 1

d *y* = 2 2 *x* + 5 + 2 4 *x* – 1

d *x* 4 *x* – 1

d *y* = 4 *x* + 10 + 8 *x* – 2

4 *x* – 1

d *x*

d *y* = 12 *x* + 8

4 *x* – 1

d *x*

1. *y* =

3 *x* + 4 2

*x* – 5 3

Use chain rule where; d

d *x*

*u n* = *ndu*

*u n* – 1

and product rule where; d *u v*

d *x*

= *udv* + *vdu*

*u* = 3 *x* + 4 2

*d u* = 6 3 *x* + 4

*v* = *x* – 5 3

*d v* = 3

d *y* =

d *x*

*x* – 5 2

3 *x* + 4 2

3 *x* – 5 2

+ *x* – 5 3

6 3 *x* + 4

d *y* = 3

d *x*

d *y* = 3

d *x*

d *y* = 3

d *x*

*x* – 5 2

*x* – 5 2

*x* – 5 2

3 *x* + 4 2 + 2 *x* – 5 3 *x* + 4

3 *x* + 4 3 *x* + 4 + 2 *x* – 10

3 *x* + 4 5 *x* – 6

1. *y* =

2 *x* – 3 4

5 *x* + 1

Use chain rule where; d

*u n* = *ndu*

*u n* – 1

and quotient rule where; d *u*

= *v du* – *udv*

d *x*

*u* = 2 *x* – 3

*d u* = 2 *dx v* = 5 *x* + 1

*d v* = 5 *dx*

d *x v v* 2

d *y* = 4

d *x*

2 5 *x* + 1 – 5 2 *x* – 3

5 *x* + 1 2

2 *x* – 3 3

5 *x* + 1

d *y* = 4

d *x*

10 *x* + 2 – 10 *x* + 15

5 *x* + 1 2

2 *x* – 3 3

5 *x* + 1

d *y* = 4

d *x*

17

5 *x* + 1 2

2 *x* – 3 3

5 *x* + 1

d *y* = 68

d *x* 5 *x* + 1 2

2 *x* – 3 3

5 *x* + 1

d *y* = 68 2 *x* – 3 3

d *x* 5 *x* + 1 5

12. *y* = 3 *x* – 4

2 *x* + 5

Use quotient rule where; d *u*

*u*

= *v du* – *udv* , and power rule where; d

= *d u*

*u* = 3 *x* – 4



2 *u*

*d u* = 3 *dx*

*v* = 2 *x* + 5

*d v* = 1 *dx*

2 *x* + 5

d *x v v* 2 d *x*

d *y* =

2 *x* + 5

3

– 3 *x* – 4

2 *x* + 5

2 *x* + 5 2

3 2 *x* + 5 – 3 *x* – 4

2 *x* + 5 3

d *x*

d *y* =

d *x*

d *y* = 6 *x* + 15 – 3 *x* + 4

2 *x* + 5 3

d *x*

d *y* = 3 *x* + 19

d *x*

13. *y* = 3

2 *x* + 5 3

*x* – 6

3 *x* + 4

Use chain rule where; d

*u n* = *ndu*

*u n* – 1

and quotient rule where; d *u*

= *v du* – *udv*

*u* = *x* – 6

*d u* = *dx v* = 3 *x* + 4 *d v* = 3 *dx*

d *y* = 1

d *x* 3

d *x*

3 *x* + 4 – 3 *x* – 6

3 *x* + 4 2

– 2

*x* – 6 3

3 *x* + 4

d *x v v* 2

d *y* = 3 *x* + 4 – 3 *x* + 18

d *x*

3 3 *x* + 4 2 3

*x* – 6 2

3 *x* + 4

d *y* = 22

d *x*

3 3 *x* + 4 2 3

*x* – 6 2

3 *x* + 4

d *y* = 22

3

2

3 3 *x* + 4 2

3

*x* – 6

3 *x* + 4 2

d *x*

d *y* = 22

3 3

*x* – 6 2 3 3 *x* + 4 4

d *x*

14. *y* = – 4 *x* – 3

3

*x* 2

Use power rule where; d *un*

d *x*

= *n un* – 1

2

*y* = *x* 3 – 4 *x* – 3

d *y* =

d *x*

1

*x* 3 – – 3  4 *x* – 4

2 –

3

d *y* = 2

3 3 *x*

d *x*

15. *y* = 4

*x*

+ 12

*x* 4

+ 1 5

Use product rule where; d *u v*

d *x*

= *udv* + *vdu* and chain rule where; d *un*

d *x*

= *ndu*

*u n* – 1

*u* = 4

*d u* = 0

*v* = + 1

*x*



1

2 *x*

*d v* = *dx*

d *y* =



4 5

1

2 *x*

*x*

10 *x* + 1 4

*x*

d *x*

d *y* =

d *x*

+ 1 4 + 0

+ 1 5

16. *y* = 4

*x*

5 *x* + 3

Use quotient rule where; d *u*

*u*

= *v du* – *udv* , and power rule where; d

= *d u*



2 *u*

*u* = 4

*d u* = 0

*v* = 5 *x* + 3

*d v* = 5 *dx*

5 *x* + 3

5 *x* + 3

2

d *x v v* 2 d *x*

d *y* =

d *x*

d *y* =

d *x*

0

–

10

5 *x* + 3

5 *x* + 3 2

5 *x* + 3

–

10

5 *x* + 3

d *y* = – 10

5 *x* + 3 3

d *x*

17. *y* =

2

4 *x* + 1 3

Use product rule where; d *u v*

d *x*

= *udv* + *vdu* and chain rule where; d *un*

d *x*

,

= *ndu*

*u n* – 1

*u* = 2

*d u* = 0

*v* = 4 *x* + 1 – 3

*d v* = – 12

d *y* = – 24

d *x*

d *y* = – 24

d *x*

d *y* = –

24

4 *x* + 1 4

d *x*

4 *x* + 1 – 4 *dx*

4 *x* + 1 – 4 + 0

4 *x* + 1 – 4

4 *x* + 1 – 3

d *y*

Evaluate d *x* at the specified value of x.

18. *y* = 6

*u* = 6

*d u* = 0

3 *x* + 2 2 , *x* = 8

*v* = + 2 2

3

*x*

2 3 *x* + 2

*d v* = *dx*

d *y* = 6

d *x*

3 3 *x* 2

2 3

3 3

*x* + 2 + 0

*x* 2

3

*x*

+ 2 2

d *y* 4 3 *x* + 2

# = , *x* = 8

d *x* 3 *x* 2

4 3 8 + 2

f ' 8 =

3 8 2

4 3 8 + 2

f ' 8 =

3

64

f ' 8 = 4 2 + 2

4

f ' 8 = 4

19. *y* =

6 – *x*

*u* = 6 –

*x*

*d u* = – 1 *dx*



2 *x*

d *y* = – 1



d *x* 2 *x*

, *x* = 4

1

2 6 – *x*

d *y* = – 1

4 *x* 6 – *x*

d *x*

f ' 4 = – 1

4 4 6 – 4

f ' 4 = –

4 2

f ' 4 = – 1

8 4

f ' 4 = – 1

16

1

6 – 2

20. *y* = *x* 3 + 4 *x* – 1 , *x* = 1

d *y* = 3 *x* 4 – 4 , *x* = 1

d *x*

f ' 1 = 3

*x* 2

1 4 – 4

1 2

f ' 1 = 3 – 4

f ' 1 = – 1

21. *y* =

2 *x* – 1 3 + 4

, *x* = 2

*u* = 2 *x* – 1 3

3 *x* – 2

*d u* = 6 2 *x* – 1 2 *dx*

– 1

*v* = 4 3 *x* – 2 2

– 3

*d v* = – 6 3 *x* – 2 2

d *y* = 6

d *x*

2 *x* – 1 2 – 6

f ' 2 = 6

3 *x* – 2 3

f ' 2 = 6

f ' 2 = 6

2 2 – 1 2 – 6

4 – 1 2 – 6

3 2 – 2 3

6 – 2 3

3 2 – 6



4 3

f ' 2 = 6 9 – 6



64

f ' 2 = 54 – 3

4

f ' 2 = 213

4

Find the slope of the tangent to the curve at the given point.

d *y* = *m or s lope*

d *x*

22. *y* = 7 – *x* 2 + 4 *x* 3 , – 1, 2

d *y* = – 2 *x* + 12 *x* 2 *x* = – 1

,

d *x*

f ' – 1 = – 2 – 1 + 1 2

f ' – 1 = 2 + 12

f ' – 1 = 1 4

23. *y* = *x* + 2 *x* – 1 , 2, 3

d *y* = 1 – 2 , *x* = 2

– 1 2

d *x x* 2

f ' 2 = 1 – 2

2 2

f ' 2 = 1 – 2

4

f ' 2 = 1 – 1

2

f ' 2 = 1

2

24. *y* = 3 *x* 2 – 4 , 2, 10

*x*

d *y* = 6 *x* + 4 , *x* = 2

d *x x* 2

f ' 2 = 6 2 + 4

2 2

f ' 2 = 12 + 4

4

f ' 2 = 12 + 1

f ' 2 = 13

25. *y* = 10 – 2 *x* , 3, 2

3 *x* 9

*u* = 10 – 2 *x*

*d u* = – 1 *dx*

10 – 2 *x*

*v* = 3 *x*

*d v* = 3 *dx*

d *y* =

3 *x* –

1

10 – 2 *x*

– 3

d *x*

d *y* = –

d *x*

9 *x* 2

10 – 2 *x*

9 *x* 2

3 10 – 2 *x* + 3 *x*

10 – 2 *x*

d *y* = – 30 – 6 *x* + 3 *x*

9 *x* 2 10 – 2 *x*

d *x*

d *y* = – 30 – 3 *x*

, *x* = 3

d *x* 9 *x* 2

10 – 2 *x*

f ' 3 = – 30 – 3 3

9 3 2

10 – 2 3

f ' 3 = – 30 – 9

10 – 6

9 9

f ' 3 = – 21

81 4

f ' 3 = – 21

81 2

f ' 3 = – 21

162

f ' 3 = – 7

54

Find the values of x for which the derivative is zero. 26. *y* = *x* 3 + 4 *x* 2 – 3 *x* – 5

d *y* = 3 *x* 2 + 8 *x* – 3, d *y* = 0

d *x* d *x*

0 = 3 *x* 2 + 8 *x* – 3

Use quadratic equation to solve for x.

*x* = 2 *a*

– *b*  *b* 2 – 4 *ac*

– 8  8 2 – 4 3 – 3

2 3

– 8  64 + 36

=

=

6

– 8  100

6

=

, *a* =

3, *b*

= 8, *c*

= – 3

*x* = – 8 + 10  1

6 3

*x* = – 8 – 18  – 3

6

27. *y* = *x* 4 – 8 *x* 3 + 22 *x* 2 – 24 *x* + 9

d *y* = 4 *x* 3 – 24 *x* 2 + 44 *x* – 24, d *y* = 0

d *x* d *x*

0 = *x* 3 – 6 *x* 2 + 11 *x* – 6

GCF of -6 is -1,1,6,-6,2,-2,3,-3

To check, it must be equal to zero.

*x* 3 – 6 *x* 2 + 11 *x* – 6 = 0

1 3 – 6 1 2 + 11 1 – 6 = 0

1 – 6 + 11 – 6 = 0

– 12 + 12 = 0

0 = 0

*x* = 1

*x* – 1 *x* 2 – 5 *x* + 6 = 0

*x* – 1 *x* – 2 *x* – 3 = 0

*x* = 1

*x* = 2

*x* = 3

28. *y* '' = 12 *x* + 8 *x* – 1

d 3*y*

d *x* 3

= 12 – 8

*x* 2

12 = 8

*x* 2

12 *x* 2 = 8

*x* 2 = 8

12

*x* =

2

3



6

*x* =

3

29. *y* = *x* – 1

*x* 2 – 2 *x* + 5

*u* = *x* – 1

*d u* = *dx*

*v* = *x* 2 – 2 *x* + 5

## *d v* =

2 *x* – 2 *dx*

*x* 2 – 2 *x* + 5 – *x* – 1 2 *x* – 2

*x* 2 – 2 *x* + 5 2

d *y* =

d *x*

d *y* = *x* 2 – 2 *x* + 5 – 2 *x* 2 + 4 *x* – 2

*x* 2 – 2 *x* + 5 2

d *x*

d *y* = – *x* 2 + 2 *x* + 3

d *x*

0 = –

*x* 2 – 2 *x* + 5 2

*x* 2 – 2 *x* – 3

*x* 2 – 2 *x* + 5 2

– *x* – 3 *x* + 1 = 0

*x* 2 – 2 *x* + 5 2

*x* = 3

*x* = – 1

Find the values of x given that;

30. *y* = 2 *x* – 3 *x* – 1

d *y* = 2 + 3

and d *y* = 14

d *x*

d *x x* 2

14 = 2 + 3

*x* 2

12 = 3

*x* 2

12 *x* 2 = 3

*x* 2 = 1

4

*x* =  1

2

2 1 d *y* 1

31. *y* = *x* 3 – *x* 3 and =

d *x* 4

*y* = –

3

*x* 2

3

*x*

*u* =

3

*x* 2

*d u* = 2 *dx*

3 3 *x*

1

*v* = *x* 3

*d v* = 1 *dx*

3 3 *x* 2

d *y* = 2 – 1

3 3 *x*

3 3 *x* 2

d *x*

1 = 2 – 1

3 3 *x*

3 3 *x* 2

4

1 = 2

4 1

– 1

1  2

3 *x* 3

1

Let x 3 = u

1. *x* 3

1 = 2 – 1

4 3 *u* 3 *u* 2

1 = 6 *u* 2 – 3 *u*

4 9 *u* 3

1 = 3 *u* 2 *u* – 1

4 9 *u* 3

1 = 2 *u* – 1

4 3 *u* 2

8 *u* – 4 = 3 *u* 2

3 *u* 2 – 8 *u* + 4 = 0

3 *u* 2 – 2 *u* – 6 *u* + 4 = 0

3 *u* – 2 *u* – 2 = 0

*u* = 2

3

*u* = 2

1

*x* 3 = 2

3

Subtitute the value of u

Cube both side of the equation

1

*x* 3 = 2

*x* = 8

27

*x* = 8

32. *y* = 3 *x* 2 + 4 *x* – 1

d *y* = 6 *x* – 4

and d *y* = 11

d *x*

d *x x* 2

11 = 6 *x* – 4

*x* 2

– 4 – 11 + 6 *x* = 0

*x* 2

6 *x* 3 – 11 *x* 2 – 4

=

0

*x* 2

*x* – 2 6 *x* 2 + *x* + 2 = 0

*x* = 2

### EXERCISE 2.3

d *y*

Use the Chain Rule to find d *x* and express the final answer in terms of x. 1. *y* = *u* 2 + *u*, *u* = 2 *x* + 1

*y* = 2 *x* + 1 2 + 2 *x* + 1

d *y* = 2 2 2 *x* + 1 + 2

d *x*

d *y* = 4 2 *x* + 1 + 2

d *x*

d *y* = 8 *x* + 6

d *x*

*x*

1. *y* =

*y* =

4 *x* 2 – 1

*u* 2 – 1 , *u* = 4

*y* = 16 *x* – 1

d *y* = 16

2 16 *x* – 1

d *x*

d *y* = 8

16 *x* – 1

d *x*

1. *y* =

*y* =

*u* – 4 , *u* = *x* 2 + 4

*x* 2 + 4 – 4

3

2

3

2

*y* = *x* 2

3

2

*y* = *x* 3

d *y* = 3 *x* 2

d *x*

2

3

4. *y* = 2 *u* – 2 , *u* = 4 *x* 3 + 1

*y* = 2 4 *x* 3 + 1 – 2

2

3

2

3

*y* = 8 *x* 3

*y* = 4 *x* 2

d *y* = 8 *x*

d *x*

5. *y* = *u* + 2 , *u* = 4 *x* – 2

*y* =

4 *x* – 2 + 2

*y* = 2

*x*

d *y* = 1



*x*

d *x*

6. *y* = 2 *u* , *u* = *x* 2

*u* 2 – 1

= 2 *x* 2

*y x* 4 – 1

*v* = 2 *x* 2

*d v* = 4 *xdx w* = *x* 4 – 1

*d w* = 4 *x* 3 *dx*

4 *x x* 4 – 1 – 2 *x* 2 4 *x* 3

*x* 4 – 1 2

d *y* =

d *x*

d *y* = 4 *x* 5 – 4 *x* – 8 *x* 5

d *x*

d *y* = –

d *x*

( x 4 -1 ) 2

4 *x x* 4 + 1

( x 4 -1 ) 2

7. *y* = *u* , *u* =

*x*

*y* =



*x*

*y* =

4

*x*

d *y* = 1

4 4 *x* 3

d *x*

d *y*

Use the Inverse Function Rule to find d *x*

8. *x* = *y* + *y* 2 + *y* 3

d *x* = 1 + 2 *y* + 3 *y* 2

d *y*

d *y* = 1

d *x*

1. *x* =

d *x* =

d *y*

d *x*



1

2 *y*

+

1

3 3 *y* 2

3 3 *y* 2 + 2 *y*

6 6 *y* 7

3 3 *y* 2 + 2 *y*

6 6 *y* 7

1 + 2 *y* + 3 *y* 2

+

*y*

3

*y*

d *y* =

d *y* =

d *x*

3

4 – 3 *y* 2

9 4 – 3 *y*

1. *x* =

d *x* = –

d *y* 2

d *y* = – 2

9 4 – 3 *y*

d *x*

11. *x* = 2

d *x* = 24

d *y*

1

24 4 *y* + 1 2

d *y* =

d *x*

1. *x* =

6

3 *y* + 1 2

*u* = 6

*d u* = 0

4 *y* + 1 3

4 *y* + 1 2

*v* = 3 *y* + 1 2

*d v* = 6 3 *y* + 1 *dy*

d *x* = 0

d *y*

3 *y* + 1 2 – 6 6 3 *y* + 1

3 *y* + 1 4

d *x* = – 36 3 *y* + 1

d *y* 3 *y* + 1 4

d *x* = – 36

d *y*

d *y* = –

d *x*

1. *x* =

*u* =

1 + *y*

1

4 1 + *y y*

## *d u* =

3 *y* + 1 3

3 *y* + 1 3

36

1 + 1 + *y*

## *dy*

d *x* = 1

8 1 + 1 + *y*  1 + *y*  *y*

d *y*

d *y* = 8  

1 + 1 + *y*

1 + *y*

*y*

d *x*

4

2 *y* + 1

3 *y* – 1

1. *x* =

*u* = 2 *y* + 1 *d u* = 2 *dy v* = 3 *y* – 1 *d v* = 3

2 3 *y* – 1 – 3 2 *y* + 1 2 *y* + 1

4

3 *y* – 1

4

3 *y* – 1 2

6 *y* – 2 – 6 *y* – 3

3 *y* – 1 2

2 *y* + 1 3

3 *y* – 1

–

20 2 *y* + 1 3

3 *y* – 1 2

3 *y* – 1

3

– 20 2 *y* + 1

3 *y* – 1 5

5

– 3 *y* – 1

20 2 *y* + 1 3

3

d *x* =

d *y*

d *x* =

d *y*

d *x*

d *y* =

d *x* =

d *y*

d *y* =

d *x*

### EXERCISE 2.4

Find the first and second derivative of each of the following; 1. *y* = *x* 5 + 3 *x* – 2 + 4 *x*

d *y* = 5 *x* 4 – 6 + 4

d *x*

d 2*y*

d *x* 2

*x* 3

= 20 *x* 3 + 18

*x* 4

2. *y* = 1

*x*

d *y* = – 1

d *x x* 2

d 2*y* = 2

d *x x* 3

3. *y* = 4 – *x* 2

d *y* = – *x*

4 – *x* 2

d *x*

*u* = *x*

*d u* = *dx v* =

4 – *x* 2

*d v* = – *x dx*

4 – *x* 2

*x* 2

d 2*y*

= –

d *x* 2

d 2*y* =

–

d *x* 2

d 2*y* =

–

d *x* 2

4 – *x* 2 +

4 – *x* 2

4 – *x* 2

4 – *x* 2 + *x* 2

4 – *x* 2

4 – *x* 2

4

4 – *x* 2 2

4. *y* = 4 *x*

+

*x* 1

*u* = 4 *x*

*d u* = 4 *dx v* = *x* + 1 *d v* = *dx*

d *y* =

4 *x* + 1 – 4 *x*

*x* + 1 2

d *x*

d *y* = 1

*x* + 1 2

d *x*

d 2*y* 2

= –

*x* + 1 3

d *x* 2

5. *y* = *x* + 5 2

d *y* = 2 *x* + 5

d *x*

d 2*y* =

2

d *x* 2

1 1 2

6. *y* =

*a* 2 – *x* 2

d *y* = – *a* – *x*

d *x* *x*

*u* = –

*a*

*x*



1

2 *x*

*d u* = – *dx*

*v* =

*x*



1

2 *x*

## *d v* =

– *x* – *a* – *x*

2 *x* 2 *x*

d 2*y*

= –

d *x* 2 *x*

d 2*y* =

d *x* 2

*a*

2 *x x*

7. *y* = 1 +

*x*

*x*

*u* = 1 +

*x*



1

2 *x*

*d u* = *dx*

*v* =

*x*



1 *dx*

2 *x*

*x* – 1 + *x*

2 *x* 2 *x*

## *d v* =

d *y* =

d *x* *x*

d *y* = – 1

2 *x x*

d *x*

*u* = 1 *d u* = 0 *v* = 2 *x*

*x*

*d v* = 3 *dx*

*x*

d 2*y* =

0  2 *x x* – 3 *x*

–

d *x* 2

1. *x* 3

d 2*y* =

d *x* 2

3 *x*

4 *x* 3

d 2*y* = 3

4 *x* 2 *x*

d *x* 2

1. *y* = *x*

*x* – 1

*u* = *x*

*d u* = *dx v* =

*x* – 1

*d v* = 1

2 *x* – 1

d *y* =

*x* – 1

2 *x* – 1

*x* – 1

2 *x* – 1 – *x*

2 *x* – 1 3

d *x*

d *y* =

d *x*

## *dx*

– *x*

d *y* = *x* – 2

2 *x* – 1 3

d *x*

*u* = *x* – 2 *d u* = *dx v* = 2

*x* – 1 3

*d v* = 3

*x* – 1

2 *x* – 1 3 – *x* – 2 3 *x* – 1

4 *x* – 1 3

*x* – 1 4 – *x*

4 *x* – 1 3

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y* = 4 – *x*

4 *x* – 1 5

d *x* 2

1. *y* =

*x* 2

*x* + 1

*u* = *x* 2

*d u* = 2 *xdx v* = *x* + 1

*d v* = *dx*

d *y* = 2 *x x* + 1 – *x* 2

d *x x* + 1 2

d *y* = 2 *x* 2 + 2 *x* – *x* 2

d *x*

d *y* =

d *x*

*x* + 1 2

*x* 2 + 2 *x*

*x* + 1 2

*u* = *x* 2 + 2 *x*

*d u* = 2 *x* + 2 *dx*

*v* = *x* + 1 2

*d v* = 2

*x* + 1 *dx*

2 *x* + 2 *x* + 1 2 – 2 *x* + 1 *x* 2 + 2 *x*

*x* + 1 4

*x* + 1 2 *x* + 2 *x* + 1 – 2 *x* 2 + 2 *x*

*x* + 1 4

2 *x* 2 + 4 *x* + 2 – 2 *x* 2 – 4 *x*

*x* + 1 3

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y* = 2

*x* + 1 3

d *x* 2

1. If y= 3

*y* =

3

*x*

d *y* = 1

3 3 *x* 2

d *x*

f ' 8 =

1

3 3 8 2

=

1

3 3 64

f ' 8 = 1

12

d 2*y*

9 3 *x* 5

= –

*x* , find f ' 8 and f '' 8 .

2

d *x* 2

d 2*y* = 2

9 *x* 3 *x* 2

–

d *x* 2

f '' 8 = –

2

9 8 3 8 2

= – 2

72 3 64

f '' 8 = – 1

144

1. If y=x 5 , find y 4 and y 4 .

*y* = *x* 5

*y* 4 = *x* 20

*y* = *x* 5

d *y* = 5 *x* 4

d *x*

d 2*y*

d *x* 2

d 3*y*

d *x* 3

= 20 *x* 3

= 60 *x* 2

d 4*y* =

d *x* 4

120 *x*

1. Find the point on the curve *y* = *x* 3 + 3 *x* for which *y*' = *y*'' *y* = *x* 3 + 3 *x*

d *y* = 3 *x* 2 + 3

d *x*

d 2*y* =

6 *x*

d *x* 2

3 *x* 2 + 3 = 6 *x*

*x* 2 + 1 = 2 *x*

*x* 2 – 2 *x* + 1 = 0

*x* – 1 *x* – 1 = 0

*x* = 1

f ' 1 = 3 *x* 2 + 3

= 3 1 2 + 3

= 3 + 3

f ' 1 = 6

f '' 1 = 6 *x*

= 6 1

f '' 1 = 6

*y*' = *y*''

*y* = *x* 3 + 3 *x*

= 1 3 + 3 1

= 4

1, 4

1. How fast does the slope of the curve *y* =

*x* 2 + *x* + 1 2

change at the point where x=2?

*y* = *x* 2 + *x* + 1 2

d *y* = 2 2 *x* + 1

d *x*

d *y* = 4 *x* + 2

d *x*

*x* 2 + *x* + 1

*x* 2 + *x* + 1

d *y* = 4 *x* 3 + 4 *x* 2 + 4 *x* + 2 *x* 2 + 2 *x* + 2

d *x*

d *y* = 4 *x* 3 + 6 *x* 2 + 6 *x* + 2

d *x*

f ' 2 = 4

2 3 + 6

2 2 + 6 2 + 2

f ' 2 = 4 8 + 6 4 + 6 2 + 2

f ' 2 = 32 + 24 + 12 + 2

f ' 2 = 70

1. Find the rate of change of the slope of the curve *y* = *x* 3 – 1 at 2, 7

*y* = *x* 3 – 1

d *y* = 3 *x* 2

d *x*

f ' 2 = 3 2 2

f ' 2 = 12

d *y*

Find d *x* by implicit differentiation. 1. *x* 3 + *y* 3 – 6 *xy* = 0

3 *x* 2 + 3 *y* 2 d *y* – 6 *x* d *y* – 6 *y* = 0

d *x* d *x*

d *y* 3 *y* 2 – 6 *x* = 6 *y* – 3 *x* 2

d *x*

d *y* = 6 *y* – 3 *x* 2

d *x* 3 *y* 2 – 6 *x*

d *y* = 2 *y* – *x* 2

d *x y* 2 – 2 *x*

2. *x* 2 + *x y* 2 + *y* 2 = 1

2 *x* + 2 *xy* d *y* + *y* 2 + 2 *y* d *y* = 0

d *x* d *x*

d *y* 2 *xy* + 2 *y* = – *y* 2 – 2 *x*

d *x*

d *y* = – *y* 2 – 2 *x*

d *x* 2 *y x* + 1

d *y* = –

d *x*

3.

*x* + *y*

1 + d *y*

*y* 2 + 2 *x*

2 *y x* + 1

+ *xy* = 21

d *x* + *x* d *y* + *y* = 0

2 d *x*

*x* + *y*

1 + d *y* + 2 *x x* + *y* d *y*

d *x*

2 *x* + *y*

d *x* = – *y*

1 + d *y*

2 *x* + *y*

2 *x x* + *y* + 1

d *x* 2 *x* + *y*

d *y* d *x* d *y*

d *x*

2 *x x* + *y* + 1

2 *x* + *y*

2 *x x* + *y* + 1

2 *x* + *y*

= – *y* +

= – *y*

1

2 *x* + *y*

= – 2 *y x* + *y* + 1

d *y* = – 2 *y x* + *y* + 1

d *x*

2 *x* + *y*

2 *x* + *y*

2 *x* + *y* 2 *x x* + *y* + 1

d *y* = – 2 *y x* + *y* + 1

d *x* 2 *x x* + *y* + 1

4. +

*x*

*y*

1 + 1



2 *x*



2 *y* d *x*

= *a*

d *y* = 0

1 d *y* = – 1



2 *x*

2 *y* d *x*

d *y* = – *y*

d *x* *x*

5. *b* 2 *x* 2 + *a* 2 *y* 2 = *a* 2 *b* 2

2 *b* 2 *x* + 2 *a* 2 *y* d *y* = 0

d *x*

1. *a* 2 *y* d *y* = – 2 *b* 2 *x*

d *x*

d *y* = – *b* 2 *x*

d *x a* 2 *y*

6. *x* – *y* 3 = *x* + *y* 2

d *y*

d *x*

d *y*

d *x*

3 1 –

*x* – *y* 2 = 2 1 +

*x* + *y*

3 *x* – *y* 2 – 3

*x* – *y* 2 d *y* = 2 *x* + *y* + 2 *x* + *y*

d *x*

d *y*

d *x*

1. *x* – *y* 2 d *y* + 2 *x* + *y*

d *y*

d *x*

d *x*

= 3 *x* – *y* 2 – 2 *x* + *y*

d *y*

d *x* 3

d *y* =

d *x*

*x* – *y* 2 + 2 *x* + *y* = 3

*x* – *y* 2 – 2 *x* + *y*

7. *y* = 4

3 *x* – *y* 2 – 2 *x* + *y*

3 *x* – *y* 2 + 2 *x* + *y*

*x* 2 + *y* 2

*y* = 4 *x* 2 + 4 *y* 2

d *y* = 8 *x* + 8 *y* d *y*

d *x* d *x*

d *y* – 8 *y* d *y* = 8 *x*

d *x* d *x*

d *y* 1 – 8 *y* = 8 *x*

d *x*

d *y* = 8 *x*

d *x* 1 – 8 *y*

8. *y* 2 = 3 *x* + 1

–

2 *x* 3

*u* = 3 *x* + 1

*d u* = 3 *dx*

*v* = 2 *x* – 3

*d v* = 2 *dx*

2 *y* d *y* = 3 2 *x* – 3 – 2 3 *x* + 1

d *x* 2 *x* – 3 2

d *y* = 6 *x* – 9 – 6 *x* – 2

d *x* 2 *y* 2 *x* – 3 2

d *y* = – 11

d *x* 2 *y* 2 *x* – 3 2

9. *y* 2 – 3 *x* + 2 *y* = 0

2 *y* d *y* – 3 + 2 d *y* = 0

d *x* d *x*

d *y* 2 *y* + 2 = 3

d *x*

d *y* =

3

2 *y* + 1

d *x*

Find y'' in each of the following.

11. *xy* = 32

*x* d *y* + *y* = 0

d *x*

*x* d *y* = – *y*

d *x*

d *y* = – *y*

d *x* *x*

2 *x* d *y* – *y*

d *y* = – d *x*

d *x* 2

*x* 2

*x* – *y* – *y*

*x*

d 2*y*

= –

d *x* 2 *x* 2

d 2*y* = 2 *y*

d *x* 2 *x* 2

2 2 2

12. *x* 3 + *y* 3 = *a* 3

2 + 2  d *y* = 0

3 3 d *x*

3 *x*

3 *y*

2  d *y* = – 2

3 3 *y*

3 3 *x*

d *x*

d *y*

3 *y*

3 *x*

d *x* = –

3 *x*

1

3

 –

3

3

*y* 2

3

*y*

*x*

– 3 *y*

1

3

3

*x* 2

3 *x* 2

2

d *y* = –

d *x* 2

–

–

3

– *y*

3 *y* 3 3 *x* 2

3

3

*x* 2

3

3 *x* 2

*y* 2 + 3 *y y*

3

9 *y* 3 *x* 2

3 *x* 2

3 *x* 2 *y* 2 + *y* 3 *y*

3 *y* 3 *x* 4

3 *y*

3 *y* 3 *x* 4

3 *x* 2 *y* + *y*

3 *y* 2

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y*

=

d *x* 2

d 2*y* =

d *x* 2

d 2*y* =

3 *x* 2 *y* + *y*

3 3 *x* 4 *y* 2

d *x* 2

13. *y* 2 – 16 *x* = 0

2 *y* d *y* – 16 = 0

d *x*

2 *y* d *y* = 16

d *x*

d *y* = 8

d *x*

d 2*y*

d *x* 2

d 2*y*

d *x* 2

d 2*y*

d *x* 2

*y*

d *y*

= – 8 d *x*

*y* 2

8 *y*

8

= –

*y* 2

= – 64

*y* 3

14. *x* 2 – 2 *xy* + 3 *y* 2 = 4

2 *x* – 2 *x* d *y* – 2*y* + 6 *y* d *y* = 0

d *x* d *x*

– 2 *x* d *y* + 6 *y* d *y* = 2 *y* – 2 *x*

d *x* d *x*

d *y* 6 *y* – 2 *x* = 2 *y* – 2 *x*

d *x*

d *y* = *y* – *x*

d *x*

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

d 2*y* =

d *x* 2

3 *y* – *x*

3 *y* – *x* 2

3 *y* – *x* d *y* – 1 – *y* – *x* 3 d *y* – 1

d *x*

d *x*

3 *y* – *x* 2

3 *y* d *y* – *y* – *x* d *y* + *x* – 3 *y* d *y* – *y* – 3 *x* d *y* + *x*

d *x* d *x*

d *x*

d *x*

3 *y y* – *x*

3 *y* – *x*

3 *y* – *x* 2

– *y* – *x y* – *x*

3 *y* – *x*

+ *x* – 3 *y*

3 *y* – *x* 2

*y* – *x*

3 *y* – *x*

– *y* – 3 *x*

*y* – *x*

3 *y* – *x*

+ *x*

3 *y y* – *x* – 3 *y* – 3 *x y* – *x* + 3 *x* – 3 *y y* – *x* + *y* + 3 *x y* – *x* – *x*

3 *y* – *x*

3 *y* – *x*

3 *y* – *x*

3 *y* – *x*

d 2*y* = 2 *x* – 2 *y*

d *x* 2 3 *y* – *x* 2

d 2*y* = 2 *x* – *y*

d *x* 2 *x* 2 +39*yy*– 2*x* =2 36

15. 4

8 *x* + 18 *y* d *y* = 0

d *x*

18 *y* d *y* = – 8 *x*

d *x*

d *y* = – 4 *x*

d *x* 9 *y*

*u* = 4 *x*

*d u* = 4 *dx v* = 9 *y*

*d v* = 9 d *y*

d *x*

d 2*y*

= –

d *x* 2

d 2*y*

= –

d *x* 2

36 *y* – 36 *x* d *y*

81 *y* 2

36 *y* – 36 *x* – 4 *x*

9 *y*

81 *y* 2

d *x*

+ 144 *x* 2

d 2*y*

d *x* 2

d 2*y*

d *x* 2

d 2*y*

d *x* 2

d 2*y*

d *x* 2

36 *y*

= – 9 *y*

81 *y* 2

= – 144 *x* 2 + 324 *y* 2

729 *y* 3

= – 9 16 *x* 2 + 36 *y* 2

9 81 *y* 3

= – 16 *x* 2 + 36 *y* 2

81 *y* 3

Find the slope of the curve at the given point.

16. 2 *x* 3 + 2 *y* 3 = 9 *xy at* 2, 1

6 *x* 2 + 6 *y* 2 d *y* = 9 *x* d *y* + 9 *y*

d *x* d *x*

6 *y* 2 d *y* – 9 *x* d *y* = 9 *y* – 6 *x* 2

d *x* d *x*

d *y* 6 *y* 2 – 9*x* = 9 *y* – 6 *x* 2

d *x*

d *y* = 9 *y* – 6 *x* 2

d *x* 6 *y* 2 – 9 *x*

d *y* = 3 *y* – 2 *x* 2

d *x* 2 *y* 2 – 3 *x*

3 1 – 2 2 2

f ' 2, 1 =

3 1 2 – 2

f ' 2, 1 = 3 – 8

–

3 2

f1'7.2*y*,31= =*x* 2––51 *at* 3, 2

3 *y* 2 d *y* = 2 *x*

d *x*

d *y* = 2 *x*

d *x* 3 *y* 2

f ' 3, 2 = 2 3

3 2 2

f ' 3, 2 = 6

12

f ' 3, 2 = 1

18. *x* 2 + 4 2 + *y* 2 = 25 *at* 4, 1

*x y*

2 *x* d *y* + *y*

2 *x* + d *x* + 2 *y* d *y* = 0

*x y*

d *x*

d *y*

1. *x* d *x*



*x y*

*x y*



d *y* 2 *x*

d *x x y*

+ 2 *y*

+ 2 *y* d *y* = – 2 *x*

d *x*



+ 2 *y* = – 2 *x* –

2 *y*

*x y*

d *y* 2 *x* + 2 *y x y* = – 2 *x x y* + 2 *y*

d *x*

*x y*

*x y*

d *y* = – *x x y* + *y* 2 *x y*

d *x*

*y x y* + *x* 2 *x y*

d *y* = – *x x y* + *y*

d *x*

*y x y* + *x*

f ' 4, 1 = – 4 4 1 + 1

1 4 1 + 4

f ' 4, 1 = – 4 4 + 1

4 + 4

f ' 4, 1 = – 9

6

f ' 4, 1 = – 3

2

3 + 2 3

19. *x*

*x y* + *y* = 9 *at* – 1, 2

3 *x* 2 + *x* 2 d *y* + 2 *xy* + 3 *y* 2 d *y* = 0

d *x*

d *y x* 2 + 3 *y* 2

d *x*

d *x*

= – 3 *x* 2 + 2 *xy*

d *y* = – 3 *x* 2 + 2 *xy*

d *x x* 2 + 3 *y* 2

3 – 1 2 + 2 – 1 2

f ' – 1, 2 = –

– 1 2 + 3 2 2

f ' – 1, 2 = – 3 – 4

+

1 12

f ' – 1, 2 = 1

13

20. + = 5 *at* 3, 2

3 *x*

3

4 *y*

3

3

2 *x*

4 d *y*

+  = 0

3 3 *y* 2 d *x*

4  d *y* = –

3

2 3 *x*

3

1. 3 *y* 2 d *x*

d *y* = –

3

27

*y* 2

2 *x* 3 4

d *x*

f ' 3, 2 = –

3

27 2

2

2 3 3 4

f ' 3, 2 = –

3

27 4

2 3 3 4

27

12

f ' 3, 2 = –

f ' 3, 2 = – 3

2 2

1. A circle is drawn with its center at (8,0) and with radius r such that the circle cuts the ellipse x

Find the radius of the circle.

+4y 2 =16 at right angles.

Equation of the circle

*x* – 8 2 + *y* 2 = *r* 2

Make the Ellipse Equation into standard form: *x* 2 + 4 *y* 2 = 16

*x* 2 + 4 *y* 2 = 16

Make the Ellipse Equation equal to 1;

16 16 16

*x* 2 + *y* 2 =

1

16 4

Differentiate both equation

*x* – 8 2 + *y* 2 = *r* 2

2 *x* – 8 + 2 *y* d *y* = 0

d *x*

2 *y* d *y* = – 2 *x* – 8

d *x*

d *y* = – *x* – 8

d *x y*

*y*' = – *x* – 8

*y*

*x* 2 + *y* 2 =

1

16 4

*x* + *y* d *y* = 0

8 2 d *x*

*y* d *y* = – *x*

2 d *x* 8

d *y* = – *x*

d *x* 4 *y*

*y*' = – *x*

4 *y*

*x* – 8  *x* = – 1,

*y* 4 *y*

*x x* – 8 = – 1

4 *y* 2

*x x* – 8 = – 4 *y* 2

Find y 2 using the ellipse equation

*x* 2 + *y* 2 =

1

16 4

*y* 2 *x* 2

–

=

1

4 16

*y* 2 = 4 *x*

–

2

4

*y* 2 = 4 *x*

–

2

4

Substitute y 2

*x x* – 8 = – 4 4 –

*x* 2

4

*x x* – 8 = – 16 + *x* 2

– *x* 2 + *x* 2 – 8 *x* = – 16

*x* = 2

*y* 2 = 4 *x*

–

2

4

2

–

*y* 2 = 4 2

4

*y* 2 = 3

Finally, find the value of r in the circle equation

*x* – 8 2 + *y* 2 = *r* 2

2 – 8 2 + 3 = *r* 2

– 6 2 + 3 = *r* 2

36 + 3 = *r* 2

*r* 2 = 39

*r* =

39

1. The vertex of the parabola *y* 2 = 9 *x* is the center of an ellipse. The focus of the parabola is an end of the minor axis of the ellipse, and the parabola and ellipse intersect at the right angles. Find the equation of the ellipse.